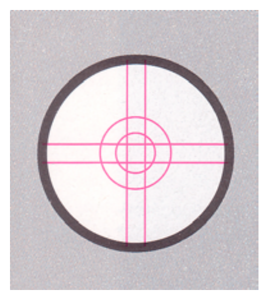
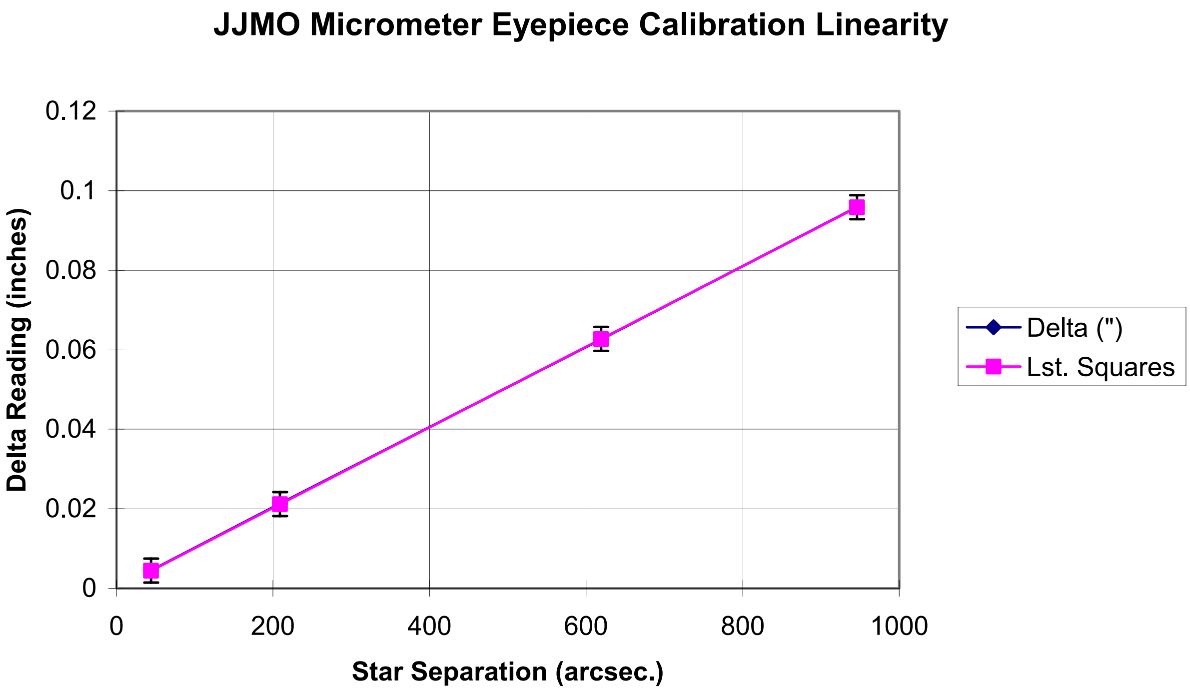
The 2003 JJMO Mars Parallax Project

The opposition with Mars on August 28, 2003 occurred only 41 hours before the Mars perihelion, reportedly the closest we have been to Mars at opposition in some 60,000 years. As described in our earlier, January 2, 2002, Journal article on the project, this was the optimum time to measure the parallax angle of Mars from Earth, in order to calculate the distance to the Sun as first done by John Flamsteed in 1672. Our measurements at the 2001 opposition, with totally insufficient instrumentation, were very crude, and led to substantial improvements in instrumentation and methodology, notably the micrometer eyepiece described in our April 25, 2002 Journal article. Since all our measurements were made during Mars retrograde motion, the technique we use to measure Mars motion in the sky (typically 20-40 arc seconds/hour during our project) was substantially improved and simplified in order to reach the required precision. Instead of timing the daily arrival of Mars in the crosshairs of a fixed telescope vs. the sidereal day, as described in the earlier report, we simply measured the shift from one of our comparison stars over a 24 hour period with the micrometer eyepiece, thus giving the motion in 24 hours to within a few arc seconds.

As in 2001, all measurements were made using the JJMO Takahashi FSQ-106mm, FL=530mm telescope and the 9mm Meade eyepiece with our addition of a Starrett micrometer head on one axis. This year we added Takahashi adapter fittings and extension tube for mounting the eyepiece securely and concentrically to the telescope. The Meade eyepiece has double cross hairs on each orthogonal axis, as shown in the adjacent figure. The vertical crosshairs were aligned with the Right Ascension coordinate by centering Mars between the cross hairs near the top (or bottom) of the Field of View (FOV), then moving the telescope in declination to the other end of the FOV. By cut and try, the eyepiece was rotated until the image kept precise alignment from top to bottom. The cross hair spacing seems to be very close to 20 arc seconds in our configuration, and since Mars' diameter during this time was about 25 arc seconds, centering was visually precise. Once this alignment was done (and checked during the night), the angular separation in RA between Mars and a comparison star was measured by centering the star between the vertical cross hairs, taking a micrometer reading, moving the cross hairs horizontally to center Mars, and taking another micrometer reading. This process was repeated until a set of 10 separations had been acquired, typically in about 8 minutes. Time was recorded to the nearest second (EDT) at each reading.



The micrometer eyepiece was calibrated each night by taking a similar set of ten readings on a pair of stars near Mars location that were separated by 9'55" as reported by our software program TheSky. In this case the "horizontal" cross hairs were carefully aligned parallel to the line between the stars, and the "vertical" cross hairs again used to measure the angular separation as described above. The calibration factor has remained remarkably stable for the past year and a half, with a variety of calibration pairs, with a value of 0.98 arc second/0.0001 inch. The best resolution with the Starrett micrometer is 0.0001 inch. We have also investigated the linearity of the micrometer eyepiece using four star doubles in Lyra, which was almost overhead during the linearity measurements to minimize refraction errors. The least squares linear curve fit to the data in the chart below was forced to pass through the origin (0,0), and indicates that the linearity of the micrometer eyepiece is more than adequate for our purposes. The error flags on the data points indicate the standard error of the measurement sets.



Diurnal parallax data was taken the nights of August 20, 28, 30, September 7, and September 16, 2003. Star separations from Mars were also taken the evenings of September 6 and 17th for estimation of Mars motion. The linearity measurements were taken the evening of September 24. In addition to cloudy nights and fog in the early morning hours, the measurements prior to the September 16-17 set were plagued with problems. Precise and reliable apparent motion of Mars was not attained until we adopted the technique described above after August 30. About that time we decided that discrepancies in the data (which were being simulated with JPL HORIZONS ephemeris data) were caused by changes in alignment of the cross hairs vs. RA during the night. The series of threaded joints in our extension tube-eyepiece adapters apparently loosened during the alignment/measurement process, and produced erratic, small misalignments that had not been noticed. Securing the threaded joints with pins through the threads solved this problem. The RA alignment procedure was also improved, with the result that our data of 9/16-17 (see spreadsheet on next page) is consistent with our JPL HORIZONS simulation, and leads to reasonably close values for the distance to Mars that night, and for the AU. The baseline we use to calculate the former from the measured parallax angle derives from the rotation of the earth during the measurements, and with the JJMO latitude of 41°31’32”N the baseline is given by:

$$\text{Baseline(miles)} = D = 5928 \cdot \sin(7.5° \cdot \Delta t(\text{hours}))$$

Mars Parallax Observations(Micrometer readings); Data of 9/16-17/2003

9/16/2003

No.	SAO164998	Time (EDT)	MARS	Delta (in.)	Delta (")
1	0.3477	20:27:26	0.3508	-0.0031	-30.4
2	0.3483	20:28:13	0.3508	-0.0025	-24.5
3	0.3483	20:29:15	0.3511	-0.0028	-27.4
4	0.3481	20:30:24	0.3505	-0.0024	-23.5
5	0.3478	20:31:16	0.35	-0.0022	-21.6
6	0.348	20:32:00	0.3507	-0.0027	-26.5
7	0.3486	20:32:49	0.3508	-0.0022	-21.6
8	0.3486	20:33:28	0.351	-0.0024	-23.5
9	0.3486	20:34:09	0.3507	-0.0021	-20.6
10	0.348	20:34:55	0.3503	-0.0023	-22.5
Angle α = -------------------->			Avg. Delta =	-0.00247	-24.2 " East
(See 1/2/2002 Journal article.)			Std. Dev. =	0.000313	3.1 "
			Std. Error =		1.0 "

9/17/2003

No.	SAO 164998	Time (EDT)	MARS	Delta (in.)	Delta (")
1	0.3606	02:31:23	0.3471	0.0135	132.3
2	0.3602	02:32:14	0.3474	0.0128	125.4
3	0.3608	02:32:58	0.3477	0.0131	128.4
4	0.3612	02:33:39	0.348	0.0132	129.4
5	0.3614	02:34:21	0.3484	0.013	127.4
6	0.3617	02:35:07	0.348	0.0137	134.3
7	0.3611	02:36:44	0.348	0.0131	128.4
8	0.3621	02:37:55	0.3488	0.0133	130.3
9	0.362	02:38:30	0.3488	0.0132	129.4
10	0.3624	02:39:19	0.3486	0.0138	135.2
11	0.362	02:39:55	0.3483	0.0137	134.3
Angle ß = ------------------>			Avg. Delta =	0.013309	130.4 " West
(See 1/2/2002 Journal article.)			Std. Dev. =	0.000324	3.2 "
			Std. Error =		1.0 "
			α-ß = 130.4 - (-24.2) =		154.6 ±2.0"

No.	SAO 164998	Time (EDT)	MARS	Delta (in.)	Delta (")
1	0.3817	20:24:33	0.3317	0.05000	490.0
2	0.3818	20:25:32	0.3309	0.05090	498.8
3	0.3813	20:26:56	0.3308	0.05050	494.9
4	0.3814	20:27:35	0.3309	0.05050	494.9
5	0.3819	20:28:24	0.3315	0.05040	493.9
6	0.3821	20:29:15	0.3311	0.05100	499.8 <----- Average time = 20:28:43.4 EDT
7	0.3817	20:30:05	0.3309	0.05080	497.8
8	0.3816	20:31:01	0.3309	0.05070	496.9
9	0.3816	20:31:35	0.331	0.05060	495.9
10	0.3822	20:32:18	0.3317	0.05050	494.9
			Average =	0.05059	495.8 " West

Calibration; Star separation = 9'55" = 595"

SAO165136	SAO165127	Delta (in.)
0.3781	0.3176	0.0605
0.3788	0.3179	0.0609
0.3792	0.3188	0.0604
0.379	0.318	0.061
0.3788	0.318	0.0608
0.3788	0.3184	0.0604
0.3791	0.3184	0.0607
0.3798	0.3191	0.0607
0.38	0.319	0.061
0.3795	0.3188	0.0607
0.3793	0.3185	0.0608
0.379	0.3186	0.0604
	Avg. Delta =	0.06072
	Std. Dev. =	0.00021
Cal. factor =		0.97991 "/0.0001 in.
(595/607.2) =		0.980 ±.003"/.0001

Transit for Mars: 9/16 @ 23:31

First meas. to Transit= 3:03:34

Transit to Last meas.= 3:08:55

Avg. time of first set: 9/16 @ 20:31:23

Avg. time of second set: 9/17 @ 02:35:39

Time interval = 6:04:16 =6.0709h

Baseline=5928·sin(6.0709·7.5°) = 4230 miles.

In 23.96 hours, Mars moved from 24.2" East of SAO 164998 to 495.8" West, or 520.0" in retrograde. So rate of motion = 21.7"/hour.

The motion of Mars during our 6.0708 h parallax measurement =21.7x6.0709 = 131.7". We label this angle "theta" = 131.7"

(See 1/2/2002 Journal article .)

Parallax = ß-α-theta = 130.4 - (-24.2) - 131.7 = 22.9 ± 2"

Distance to Mars = 4230/parallax(radians) = 4230/0.00011102 = 38.1 ±4.0 M mi.

The expression for our baseline at the JJMO as a function of the elapsed time between parallax observations is arrived at as follows:
The radius, r, from the JJMO to the spin axis of Earth is given by the average radius of Earth, 3,958.76 miles = R, multiplied by the cosine of our latitude (see Figure 1 below). In Δt hours, this radius will sweep through an angle of 2A centered on Earth's axis, where $A = 360° \times \Delta t/48 = 7.5° \cdot \Delta t$. To find the desired chord distance D between the locations of the JJMO at the early and late measurements, we solve the right triangle shown in Figure 2 below to obtain

$$D/2 = r \cdot \sin(A) = \text{average Earth radius} \cdot \cos(\text{Lat.}) \cdot \sin(7.5° \cdot \Delta t).$$

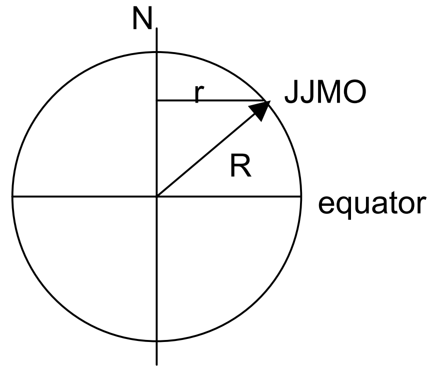


Figure 1

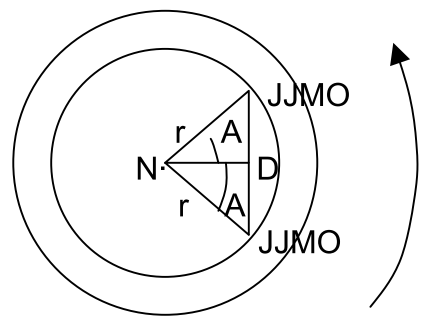
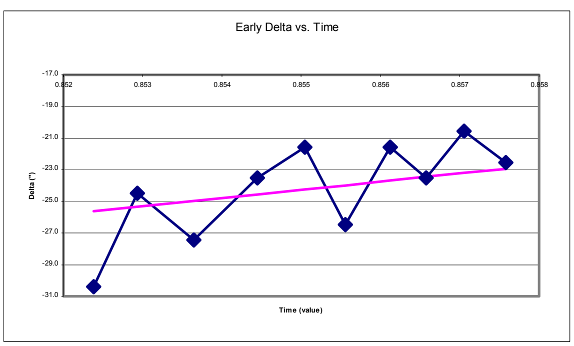
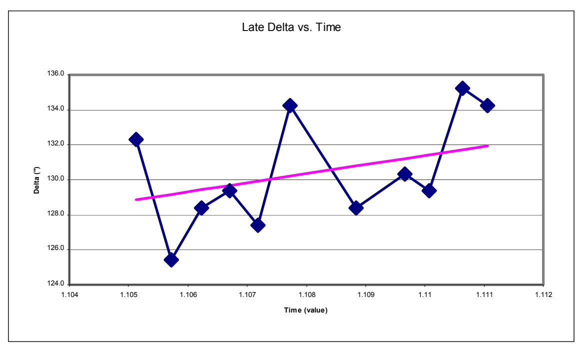
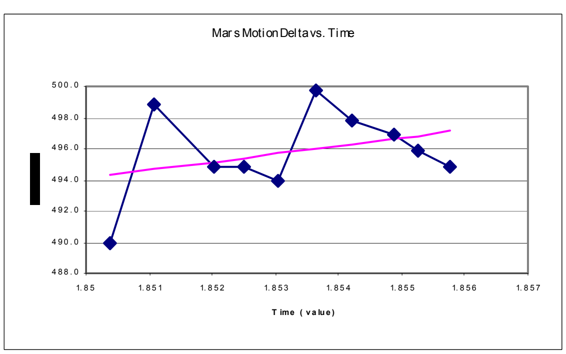


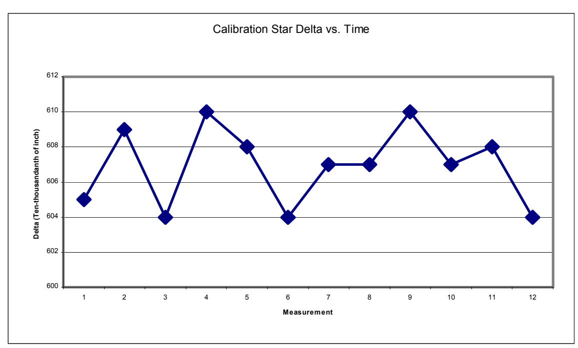
Figure 2

While not readily apparent in the numerical data for each of the four sets of observations shown in the spreadsheet, the drift of Mars during our short sets of observations is easily seen when they are plotted vs. time:









The project design included the following factors:

1. Parallax measurements were equally spaced before and after transit in order to minimize refraction errors and maintain geometric symmetry.
2. To improve the effective resolution of the micrometer eyepiece, statistical sets of ten measurements were made and averaged.
3. To minimize the effect of Mars motion relative to the star during each set of measurements, the measurements were made as quickly as possible by speaking the readings to a scribe (a tape recorder was deemed less desirable).
4. The baseline was deemed to start and end at the midpoint of the sets of measurements, which took only 7 to 8 minutes. The baseline was kept as long as possible, consistent with darkness and item 1, to maximize the observed parallax.
5. The micrometer eyepiece was calibrated each night using the same procedures used for the parallax measurements, and using a star pair near Mars to insure the same refraction effects.
6. Insofar as possible, the telescope was allowed to just track Mars between the early and later parallax measurement sets.

Approximations made:

1. The angular speed used to calculate the motion of Mars during the measurements was the speed about 9 hours after the midpoint of our measurement. Ideally, we would have measured the separation from SAO 164998 around 20:30 on 9/15, so that the average speed derived from that measurement and the one on 9/17 would have been the speed at 20:30 on 9/16, during the first set of parallax measurements, and only 3 hours before transit. But 9/15 was cloudy!

RESULTS

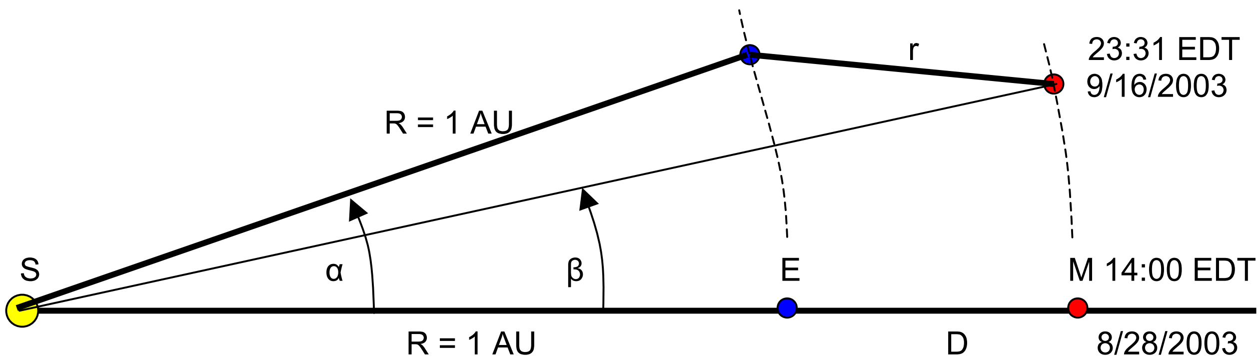
From our measured value of 22.9±2.0" for the parallax angle of Mars at the JJMO at 23:31 on 9/16/2003, and our calculated baseline of 4230 miles, we calculate the distance to Mars at that time to be ~38.1 ±4.0 million miles. The value from the JPL ephemeris is 0.4026 AU, or 37.4 M mi.

Using Kepler's Laws of planetary motion, well known to Flamsteed in 1672, it is possible to use the measured distance to Mars on 9/16/2003 to calculate the distance from Earth to Sun, known as the Astronomical Unit, or AU. This mathematical exercise is included in the Addendum which follows, but the result is 94.2±9.0 M mi..

It is a pleasure to acknowledge the help of others whose contributions made this project possible. Tony Vill fabricated the custom parts needed to convert the Meade eyepiece into a precision instrument; Monty Robson provided the inspiration and motivating force for the project as well as support during the last two years; and Amy Ziffer and Sheila McMahon performed admirably as on-time, on-call, and accurate scribes during the data taking sessions.

Parker Moreland 11/15/2003

ADDENDUM
Calculating the AU from a parallax measurement of Mars' distance from Earth near the time of a Mars perihelion opposition:



The 2003 Mars opposition with Earth was at ~14:00 EDT on 8/28/2003, and was followed 41 hours later, at ~07:00 on 8/30/2003, by Mars perihelion. The JJMO Mars Parallax Project team took parallax measurements of Mars the night of Sept. 16-17, 2003, some 19 days after opposition, and found that the distance r to Mars then was ~38 M mi. Had we been successful in measuring D with our 8/28-29 measurements, the calculation of the AU would have been simple, using Kepler's Third Law. The purpose of this note is to estimate the value of the AU using our measurement of r > D.

Assumptions:

1. Earth's orbit is circular, and its angular rate of motion is the annual average rate. Since our measurements were taken shortly before the autumnal equinox, these are valid approximations.
2. The distance from Earth to Mars, D, at perihelion opposition is related to the average distance from Earth to Sun, R, by R = 2.622·D. See Appendix 1 for this derivation using Kepler's Third Law of planetary motion.
3. The angular rate of motion of Mars near its perihelion is 1.211 times its average rate. See Appendix 2 for a derivation using Kepler's Second Law of planetary motion.
4. The Mars 2003 opposition occurred within 41 hours of perihelion (data from the JPL Horizons Ephemeris). For these calculations, perihelion and opposition are considered simultaneous.

We measured the distance to Mars, r in the figure above, on 9/16/2003, and can estimate the angles α and ß from assumptions 1 and 3. Based on assumption 2, we define $D \equiv k \cdot R$, where k = 0.3813. The solution for R in the triangle above is found using the Law of Cosines:

$$r^2 = R^2 + R^2(1+k)^2 - 2R^2(1+k)\cdot\cos(\alpha-\beta)$$

Thus,

$$R = 1\ \text{AU} = r/\sqrt{k^2 + 2(1+k)(1-\cos(\alpha-\beta))}\ .$$

Our estimates for α and ß based on assumptions 1 and 3 are:

α = [360°/365.25d]x19d = 18.7° and ß = 1.211[360°/687d]x19d = 12.1°.

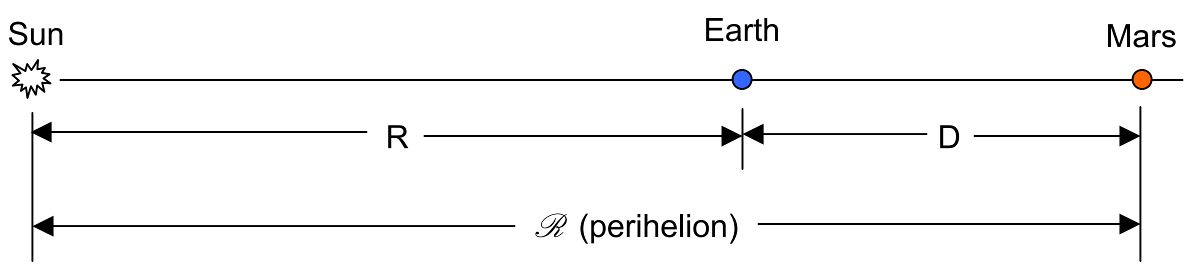
So α –ß = 6.6°.
Thus the AU, based on our 9/16-17/2003 parallax measurement of r ≈ 38.1 M mi., is

$$R = 1\text{ AU} = 38.1\text{ M mi}/\sqrt{0.3813^2 + 2\cdot 1.3813\cdot(1-\cos(6.6))}$$
$$= 94.2\text{ M mi.} \quad \sim 94\text{ M mi.}$$

<u>**Appendix 1**</u>:

<u>The relationship between the AU and the Earth-Mars distance at perihelion opposition.</u>

Let R be the radius from the Sun to Earth's orbit, $\mathscr{R}$ the perihelion radius of Mars orbit, and D the distance from Earth to Mars at opposition, as depicted in the figure below.



Since the semi-major axis of Mars orbit is $\mathscr{R}/(1\text{-}e)$, Kepler's Third Law of planetary motion, later shown by Newton to apply to all satellite orbital motions, states that for R = average Earth orbital radius = 1 A.U.:

$R^3 = kT^2$ for Earth, and $[\mathscr{R}/(1\text{-}e)]^3 = k\Im^2$ for Mars, where k is a numerical constant and

T = period of Earth's orbit = 365.25 days, $\Im$ = period of Mars orbit = 686.95 days, and
e = eccentricity of Mars orbit = 0.09341. Note that $(1\text{-}e)^3 = 0.74513$.

If we divide one equation into the other and then note that $\mathscr{R} = R + D$, we obtain

$$(\mathscr{R}/R)^3 = [(R+D)/R]^3 = [1 + (D/R)]^3 = 0.74513 \cdot (\Im/T)^2$$

Substituting the numerical values for $\Im$ and T, we find

$$\frac{D}{R} = \sqrt[3]{0.74513\left(\frac{686.95}{365.25}\right)^2} - 1 = 0.38134\text{ , so that } \boxed{R = 2.622\cdot D}$$

Appendix 2

The orbital angular speed of Mars at perihelion.

We shall need two "handbook facts" about ellipses:

1. The equation of an ellipse in polar coordinates (r,θ) is

$$r = \frac{a \cdot \left(1 - e^2\right)}{\left(1 + e \cdot \cos\theta\right)}$$

where e is the eccentricity and a is the semi-major axis, or average radius, of the ellipse.

2. The area of an ellipse is $A = \pi \cdot a^2 \sqrt{\left(1 - e^2\right)}$

The angular rate of motion in orbit can be found using Kepler's Second Law:

$$dA/dt = \text{constant} = ½\, r(ds/dt) = ½\, r^2(d\theta/dt) \equiv ½\, C.$$

Thus $\quad d\theta/dt = C/r^2$.

We can evaluate the constant C in terms of the known period, T = 687d and eccentricity, e = 0.09341, of Mars orbit as follows:

Total area of orbit = $\int dA = \pi a^2 \cdot \sqrt{\left(1 - e^2\right)} = ½\, C \cdot \int dt = ½\, C \cdot T$

Therefore,

$$C = a^2 \cdot (2\pi/T) \sqrt{\left(1 - e^2\right)}$$

And

$$\frac{d\theta}{dt} = \frac{C}{r^2} = \frac{2\pi a^2 \left(1 + e \cdot \cos\theta\right)^2 \sqrt{\left(1 - e^2\right)}}{T \cdot a^2 \left(1 - e^2\right)^2}$$

Perihelion (smallest value of r) occurs at θ =0, so at perihelion

$$\boxed{\frac{d\theta}{dt} = \frac{2\pi}{T} \cdot \frac{\left(1 + e\right)^2}{\sqrt{\left(1 - e^2\right)^3}} = 1.211 \cdot \frac{2\pi}{T} = 1.211 \cdot \text{average angular speed}}$$