The JJMO Mars Parallax Project

A. Introduction

During May of 200l, Monty Robson suggested that an interesting amateur project would be to replicate the measurements that were made in 1672 by two of the foremost astronomers of the 17th century, leading for the first time to knowledge of how truly huge the solar system is. From their measurements of the "parallax" of Mars, the first reasonably close size of the Astronomical Unit, the mean distance from Earth to Sun, was obtained. Monty felt that, since prime Mars oppositions in 2001 and 2003 were coming up, this could be an exciting project for high school students or JJMO amateurs. The 2001 opposition was on June 13, right in the middle of exams at the high school, so no student interest was forthcoming after our publicity. Monty and I therefore decided to tackle the project in 2001 for "practice", and hope that by the best Mars opposition in a 16 year cycle on August 28, 2003, we might have student involvement.
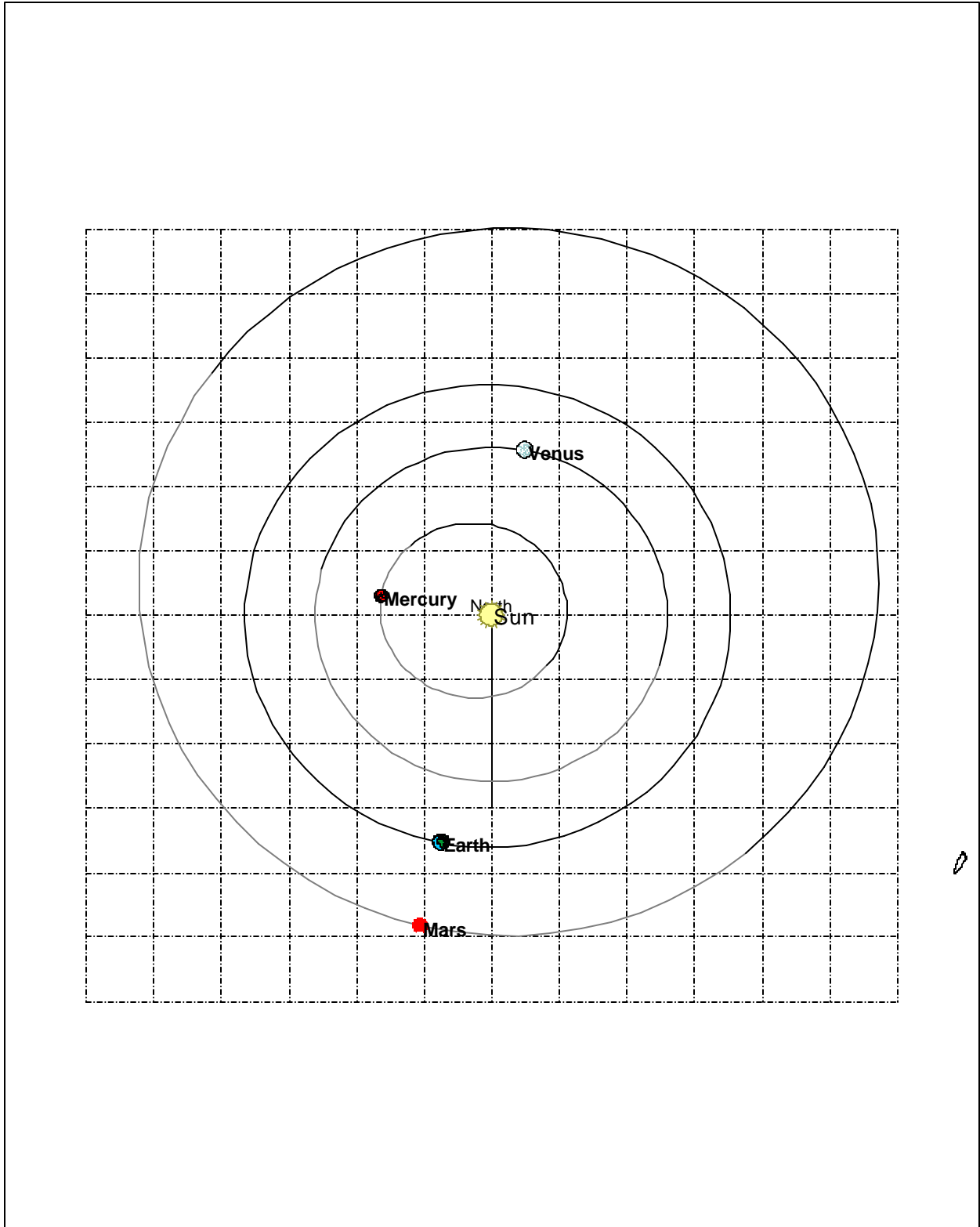
B. Historical Background

In the early 1600's Galileo and others introduced the telescope to astronomy. By 1650 improved optics for telescopes were available, and it had been discovered how to place a cross-hair at the image plane of the objective lens to facilitate measurements. And by 1670, micrometer eyepieces with precisely adjustable cross-hairs had become available. The stage was set for a new level of precision measurement in astronomy.

In 1672, Giovanni Cassini in Paris and John Flamsteed, in Derby, England, both realized two things:
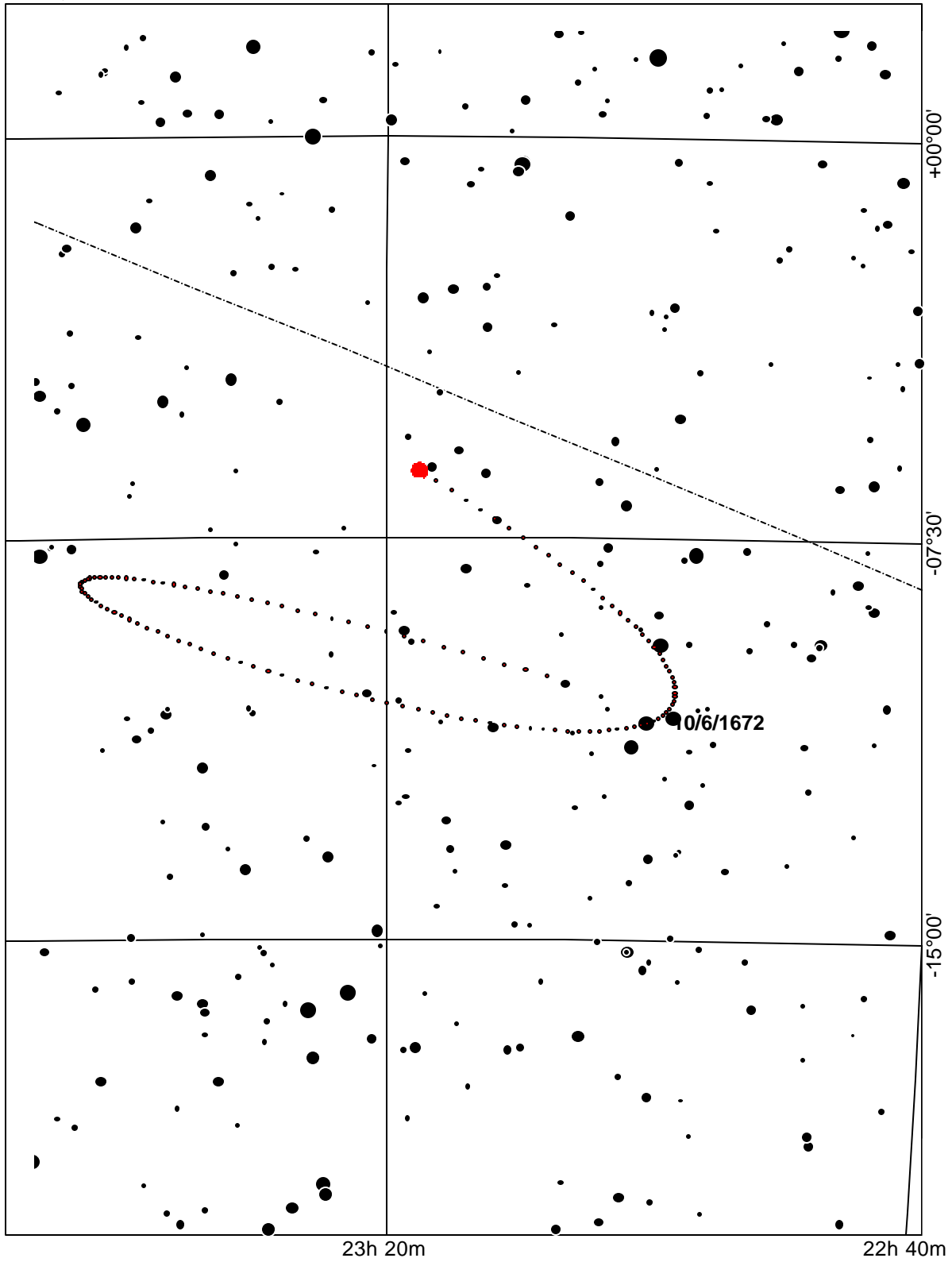
1. A measurement of the distance to Mars from Earth during the Mars opposition, obtained by measuring the parallax of Mars as viewed from Earth, would permit the calculation of the distance of Earth from the Sun at that time, closely approximating the value of the Astronomical Unit.

2. The opposition of Mars predicted for the fall of 1672 would provide an excellent opportunity to measure the Mars parallax, since Mars would be at its perihelion (point in orbit closest to the Sun) at opposition, and Mars would be passing through some bright star fields near opposition, facilitating parallax measurement with micrometer eyepieces.

An "Orrery view" of the inner planets shows the alignment at the opposition of September 9, 1672, and the somewhat elliptical nature of Mars orbit [TheSky file Orrery1672].   Using the Time-Skip feature of TheSky, we can plot the positions of Mars against the stars during the late summer and fall of 1672, and notice its close approach to several magnitude 4 stars in early October, 1672, fortuitously near the turnaround from retrograde to normal orbital motion. [TheSky file Mars1672]. Although Flamsteed

Mercury

Sun

Venus

North

Earth

Mars

"Orrery View" of inner solar system at the 9/9/1672 opposition of Mars.

TheSky file Mars1672

+00°00'

-07°30'

-15°00'

10/6/1672

23h 20m

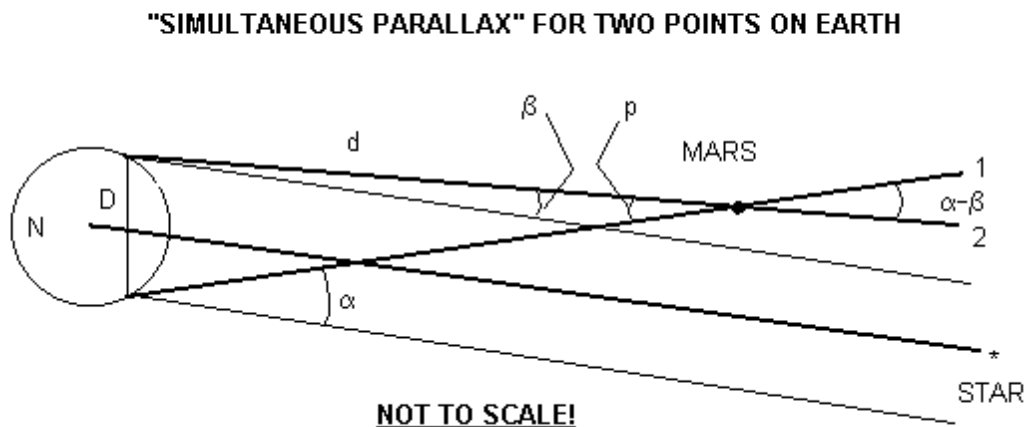22h 40m

1672 retrograde motion of Mars around the 9/9/1672 opposition.

and Cassini did not have a computer running TheSky, their intimate familiarity with the heavens allowed them to predict this trajectory.

At this point we need to define with some precision what "parallax" is and how it can be observed and measured. Just as the speed seen on an ordinary pointer-and-scale speedometer appears to be different when viewed from the passenger seat or the drivers seat, due to the spacing between the pointer and the scale, so to does the position of a nearby celestial object with respect to the very distant stars depend on the viewing position. We call this phenomenon parallax. The figure below depicts (NOT TO SCALE) two lines of sight [1 and 2] from Earth to Mars near the Mars opposition. Using a much more distant star for our reference, we see that Mars APPEARS to be separated from the star by an angular separation α from position 1, and by an angular separation β from position 2. We will define the parallax angle to be angle p in the diagram, and notice from geometry that angle $p = \alpha-\beta$, since the dotted lines, representing the direction of the very distant star, are parallel. If d is the distance from Earth to Mars, and D is the separation between the two observing points on Earth, we should be able to use geometry to calculate d if we know D and can measure angle p (see section C below.).

"SIMULTANEOUS PARALLAX" FOR TWO POINTS ON EARTH

β p

d   MARS

N   D   1

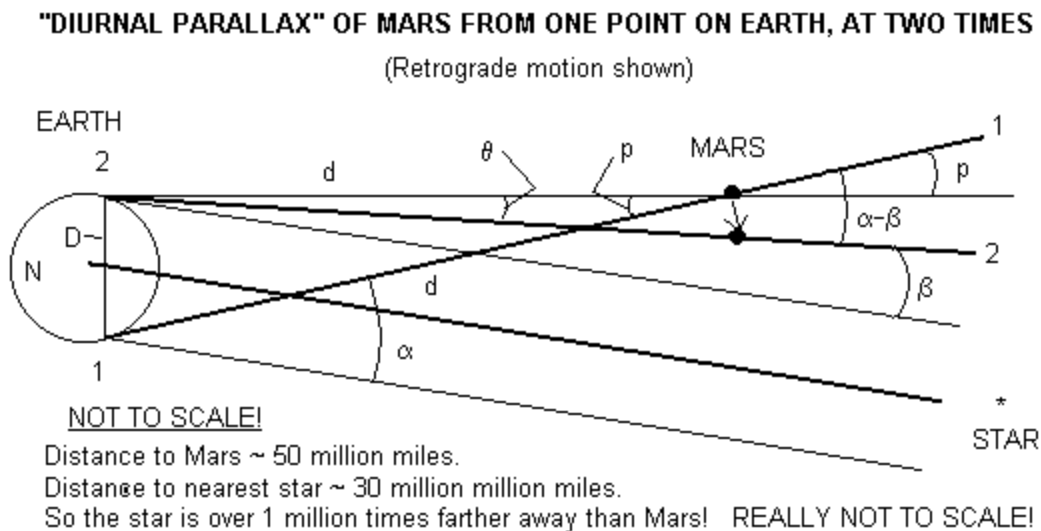$\alpha-\beta$

2

α

*   STAR

NOT TO SCALE!

Even with micrometer eyepieces, Cassini and Flamsteed knew that the small angle p would be very hard to measure (the parallax for Mars turns out to be just a few arc-seconds!). In order to improve the chances for success, d should be as small as possible (i.e., Mars should be at perihelion, as near to Earth orbit as possible), and D should be as large as possible (but no larger than Earth's diameter!). The opposition of 1672 insured a d at perihelion. But Cassini and Flamsteed used different techniques to obtain a large observation separation D. Cassini sent a colleague to French-controlled Cayenne Island off the coast of South America and just North of the equator about a year before the Mars

opposition, with instructions to make a number of astronomical measurements, including the exact latitude and longitude of the island so that D could be calculated, and the angular separation of Mars from a specified bright star at a specified time. Cassini measured the same angular separation at the same time in Paris, and then waited nearly a year for his colleague to return with his data!

Flamsteed decided to make all measurements himself in one night at Derby, taking one sighting early in the evening (sighting 1) and another the next morning (sighting 2) after the Earth had rotated to create the observation separation D. But the price to be paid for such a solo performance (which insures that the same experimental bias exists for both measurements, made by one individual with one instrument) is that Mars moves in its orbit during the hours between the two measurements. And the angular distance Mars moves is usually MUCH larger than the sought-after parallax angle. So a series of measurements must also be made to measure precisely the motion of Mars in Right Ascension (RA) at the time of parallax measurements.

The measurement of parallax from one location is called "diurnal parallax". Diagrammatically, we have the following situation:

"DIURNAL PARALLAX" OF MARS FROM ONE POINT ON EARTH, AT TWO TIMES
(Retrograde motion shown)

NOT TO SCALE!
Distance to Mars ~ 50 million miles.
Distance to nearest star ~ 30 million million miles.
So the star is over 1 million times farther away than Mars!   REALLY NOT TO SCALE!

Here α is the angular separation between Mars and the comparison star for observation 1, β is the angular separation for later observation 2, and θ is the angular "proper motion" distance Mars has moved in RA during the time between observations 1 and 2. Using some geometry on the triangle shown, one can show that the parallax angle is: $p = \pi - \theta - (\pi - \alpha + \beta) = \alpha - \beta - \theta$.
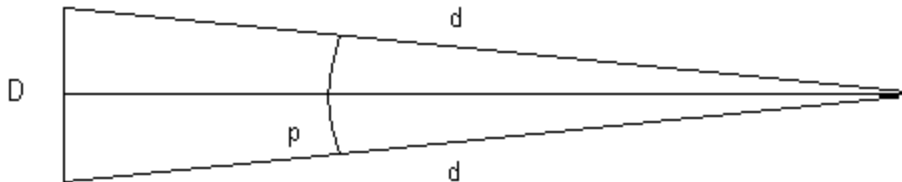
Reportedly, Flamsteed made angular separation measurements between Mars and two of the three nearby magnitude 4 stars visible on 10/6/72, probably SAO 146620 and SAO 146598.

But here we notice the luck and shrewdness of Flamsteed – on the night of October 6, 1672 when he took his measurements, Mars was virtually at its retrograde turnaround, which means that its RA motion was almost zero! So Flamsteed had very little in the way of a proper motion correction to make. Measurements taken at opposition some four weeks earlier would have had large proper motions, so that one would be subtracting large numbers to find the small parallax angle as the difference. The good experimenter must take every advantage he can in order to wrest good data from Mother Nature! Flamsteed knew it is not good to have to obtain a result that is the difference of two large numbers, each with some measurement error. The uncertainty in the result can be larger than the resulting value itself! And remember, Cassini and Flamsteed were using pendulum clocks to measure time – without motor drives on their telescopes, there was plenty of uncertainty.

In 1673, both Cassini and Flamsteed reported that, based on their measurement of the parallax of Mars, the distance from Earth to Sun was estimated to be 87,000,000 miles. For the first time, astronomers had a true vision of the size of the solar system!

C. Mathematical Background

1. <u>Finding the distance to Mars, d, from the measured parallax angle, p</u>:
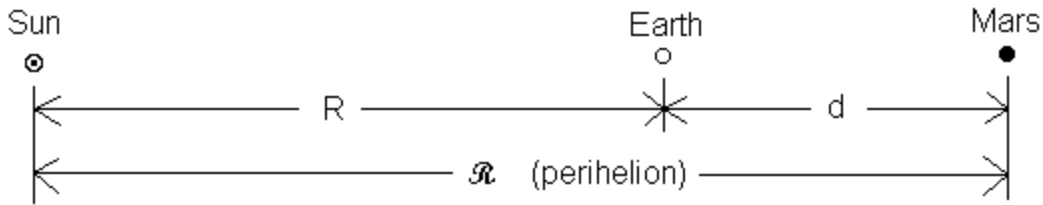

For a parallax measurement baseline D, trigonometry says

$$\sin(p/2) = (D/2)/d \text{ , so } d = D/[2 \cdot \sin(p/2)].$$

But for d >> D and thus very small angles p, it can be shown that if we measure the angle in radians,

$$\sin(p/2) \approx p/2 \text{ , and finally } d \approx D/p(\text{radians}).$$

[Note that this is equivalent to saying that for a very large radius, the baseline D is very nearly the same length as the arc length S of a circle of radius d, with $S \equiv d \cdot p(\text{rad.})$.]

2. <u>Finding the distance from Earth to Sun (the Astronomical Unit) from the measured distance to Mars at a perihelion opposition and Kepler's Third Law.</u>

Kepler's Third Law of planetary motion, later shown by Newton to apply to all satellite orbital motions, states that

$R^3 = kT^2$ for Earth, and $[\mathcal{R}/(1-e)]^3 = k\mathfrak{I}^2$ for Mars, where k is a numerical constant and

T = period of Earth's orbit = 365.25 days, $\mathfrak{I}$ = period of Mars orbit = 686.95 days, and e = eccentricity of Mars orbit = 0.09341.   Note that $(1-e)^3 = 0.74513$.

If we divide one equation into the other and then note that $\mathcal{R} = R + d$, we obtain

$(\mathcal{R}/R)^3 = [(R+d)/R]^3 = [1 + (d/R)]^3 = 1 + (d/R)^3 + 3(d/R)^2 + 3(d/R) = 0.74513 \cdot (\mathfrak{I}/T)^2$

Substituting the numerical values for $\mathfrak{I}$ and T, and letting d/R = x, we recognize the equation for a cubic polynomial

$$x^3 + 3x^2 + 3x = 1.6357.$$

This cubic equation can be solved graphically or by using other techniques. The only real solution is

x = d/R = 0.3813,   or   R = d/0.3813 = 2.622·d

So if we can measure the distance d of Mars from Earth, we can calculate the distance from Earth to the Sun, R, known as the Astronomical Unit.

D. The JJMO "Mars Parallax Project"

After we decided to attempt to replicate the 1672 measurements, Monty selected the "simultaneous parallax" method of Cassini, and I undertook the "diurnal parallax" method of Flamsteed. In preparation I purchased a Meade 9 mm Plossel eyepiece with an illuminated micrometer reticle carrying double cross-hairs. Since the adjustment knobs had no dials, I adapted a turns-counting dial with 50 divisions per turn to one of the adjustment screws, and by the week of June 27, 2001, some two weeks after the 2001 opposition, I was able to get a series of semi-clear nights and take some preliminary data on Mars-Star separations and on Mars proper motion. The procedure was as follows:
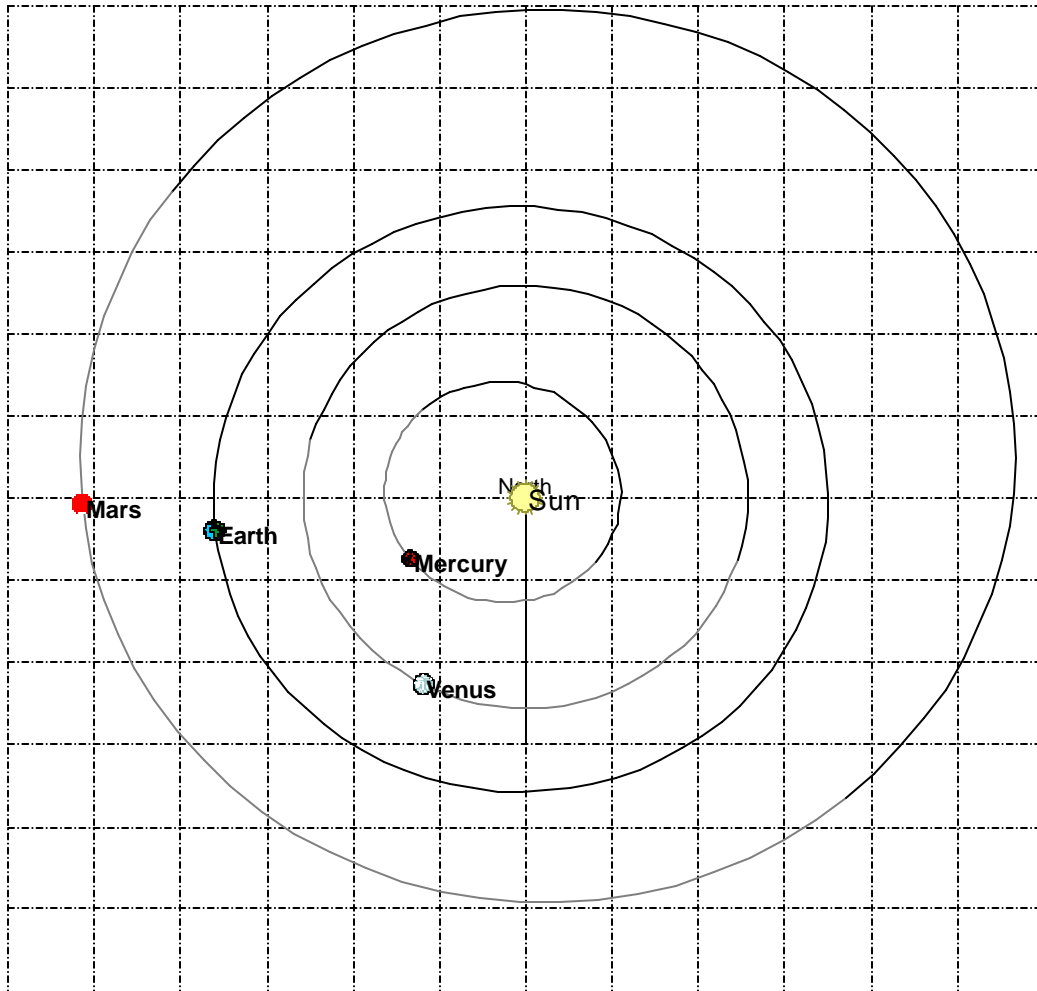
1. Using a small (90 mm, F = 1000 mm) dedicated and untouched refractor at home, equipped with Bob Lambert's unmodified micrometer eyepiece, the angular retrograde motion of Mars among the stars was measured by timing when Mars aligned with the cross-hairs each night, near the transit time of Mars. A sidereal day correction is made to obtain the angular rate, in arc-minutes/hour ('/h), for Mars. These daily measurements, which change slowly, are then graphically interpolated to the times of the parallax measurements on the night of 6/27/2001. I simply used the rate interpolated to the transit time for 6/27.

2. The measurement of the angular parallax + proper motion (angle (α - β) above) for Mars is made using the JJMO Takahashi 106 mm, F = 530 mm refractor, with the specially adapted 9 mm eyepiece. With this combination, the range of adjustment of the cross-hairs represents about 20' of angle in the sky, and takes about 4 to 5 turns of the adjusting screw. For the 6/27 measurements, the dial was simply held steady with one hand while the knob was adjusted, a reading taken, the knob adjusted again, and another reading taken – a procedure that is prone to reading errors.
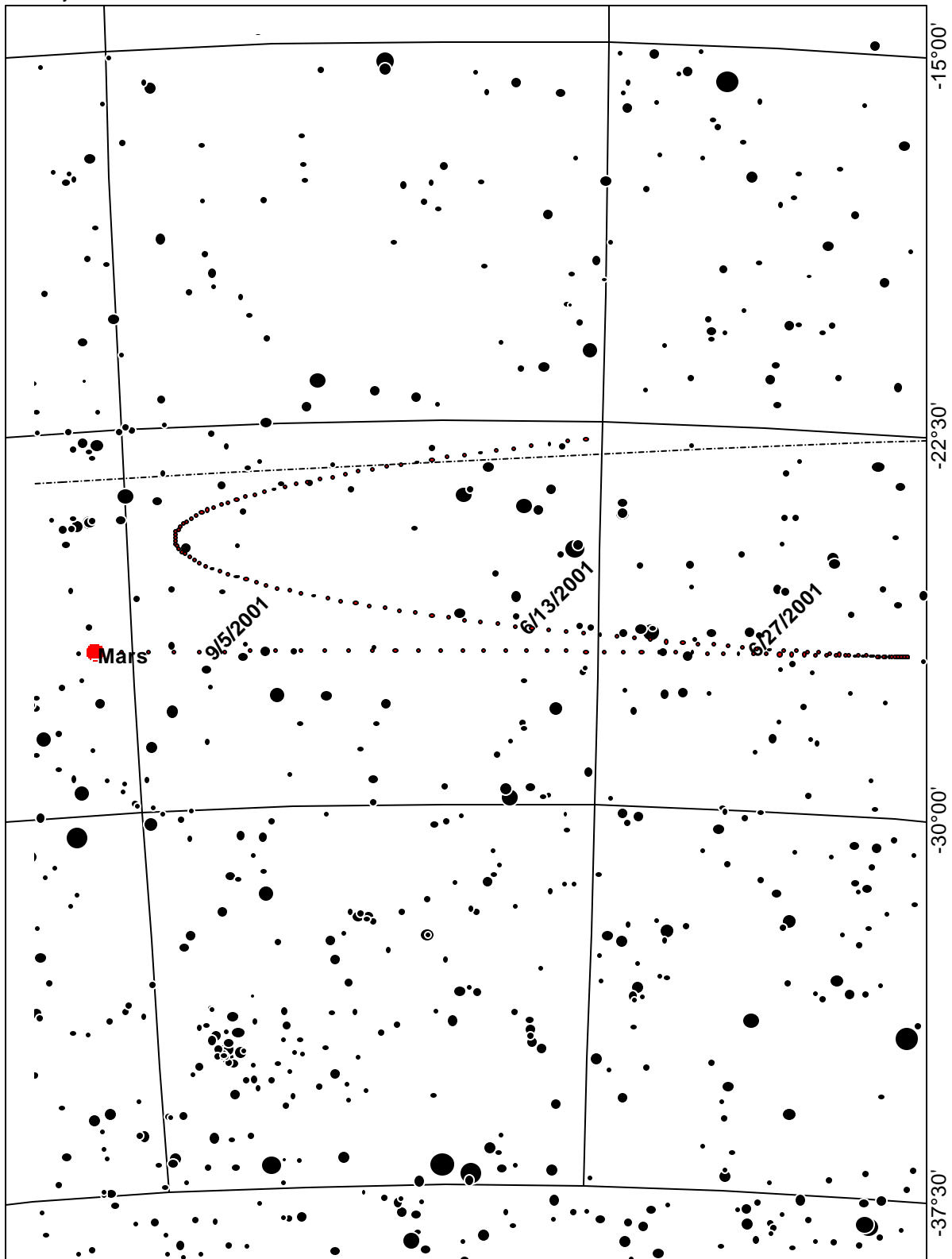
The parallax + proper motion measurement consists of using the micrometer eyepiece to measure the angular separation in RA between Mars and a star that is within the 20' range of the micrometer. This measurement (actually a series of measurements) is made at some time interval before the Mars transit, and then again at the same time interval after the transit, in order to minimize atmospheric refraction effects. The earth's rotation between the two series of measurements produces the baseline distance D, which can be calculated from the overall time elapsed, latitude of observations, and the radius of the earth.

3. The measurement procedure also includes calibrating the micrometer eyepiece by measuring the angular separation between two stars of known separation (again, < 20') and measuring the number of turns of separation to the nearest 0.01 turn. To make this measurement the measurement cross hairs must be aligned perpendicular to the line connecting the two stars, of course. After the calibration, the measurement cross-hairs are aligned parallel to the RA axis by centering on Mars, say, and then scanning in declination while rotating the eyepiece as required to make the object stay between the double cross-hairs for the entire field of view. Then one can make Mars to star separation-in-RA measurements.

 The "Orrery View" on the next page shows that the opposition of 2001 was not very well aligned with the perihelion of Mars, and on the page following that we see that by the time our jury-rigged micrometer eyepiece was ready and cloudy skies dissipated, we were forced to use a magnitude 6 star for parallax comparison measurements, and a nearby magnitude 7.5 star to calibrate the eyepiece in arcseconds.

"Orrery View" of inner solar system during the JJMO measurements of 6/27/01.

Mars

9/5/2001

6/13/2001

6/27/2001

-15°00'

-22°30'

-30°00'

-37°30'

2001 Retrograde motion of Mars, near the 6/13/01 opposition.

On the night of 6/27/01 a series of separation measurements were taken about 1 ½ hours before and after the ~11:30 PM EDT transit of Mars. Five sets that are most nearly at equal before and after times have been analyzed and are summarized in the two tables at the top of the last page. The second (center) table shows the raw micrometer dial readings and the resulting calibrated values of α, β, and their difference, while the top table uses these values and the chord and Mars proper motion, θ, resulting from the given time intervals between measurements to calculate the parallax angle, p, and its associated distance to Mars. Although 2 of the 5 measurements yield values of p within 30% of the correct value, the overall average of the 5 is nearly three times the correct value or 300% off.

To check these results, we simulated our measurements by using the Right Ascension of Mars (as observed from the JJMO at the times of our measurements) that can be obtained from the JPL HORIZONS program. By taking the difference between these RA values for Mars, and the known RA for comparison star SAO 185033, we calculated the angles α and β at the times of our data, and again using our own measurements of the Mars proper motion, θ, we obtained a simulated parallax angle based on the JPL ephemerides. The results of this simulation are shown in the bottom table on the last page. In each case, we now get consistent values of p that are close to the correct value (near 11") for 6/27/01.

Using the JPL data as a guide, one can further analyze our experimental data, and infer the following:

1. There is substantial scatter in the data, almost certainly due to the fact that the turns-counting dial was not rigidly connected to the eyepiece during the measurements. The α measurement for data point 3 is very far off, and this datum should be rejected.

2. The dial calibration appears to be too low at 278"/turn, and in fact the discrepancy seems to be larger for the β measurements (later) than the α measurements. If the calibration is changed by about 10% and 5% respectively, an average parallax close to the correct value can be obtained. Why the calibration should depend on time/direction of observation is unclear, and makes this hypothesis suspect.

Based on these preliminary results and analysis, a bracket to rigidly mount a Starrett micrometer head (with 0.0001 inch resolution) to the eyepiece has been designed and fabricated (with the help of Don Ross and Tony Vell), so that the reading precision will be about 1 arcsecond per 0.0001 inch.

Other procedural changes being adopted are:

1. Dial calibrations will be done at both sets of measurements (early and late).

2. Each data set (early and late) should have 10 or so individual measurements, and the time of each should be recorded to within 10 sec. (6/27 data was within ~ 1min.!)

3. The timing measurements for Mars proper motion in 2001 were taken from GPS readings, and are good to within about 1-2 sec. we believe. We will now make these measurements while recording the Canadian standard time station CSU (or the US station WWV) and should obtain ¼ sec. precision in this way.

Experimentation is also being done to determine if our calibration techniques are accurate and adequate. It also appears that reproducible placement of the eye to the eyepiece and rigid connection of the eyepiece to the telescope (adapters) are important.

E. The JJMO "Mars Parallax" Project – Part II

While the 6/13/01 opposition of Mars was a very good one in that it occurred near the perihelion of Mars orbit, making the parallax angle relatively larger and giving us a very bright, large object to view, the opposition of 8/28/03 should be even better. Mars will be very close to perihelion and significantly closer to Earth (about 35 million miles, vs. about 42 million in 2001), its magnitude will be -2.9 (vs. -2.3 in 2001), and the diameter of its disk viewed from Earth will be 25% larger (25 arc sec vs. 21 arc sec). In addition, Mars will be substantially higher in the sky -- altitude of 31° instead of the relatively low 21° in 2001. All in all, the viewing of Mars will be about as good as it gets.
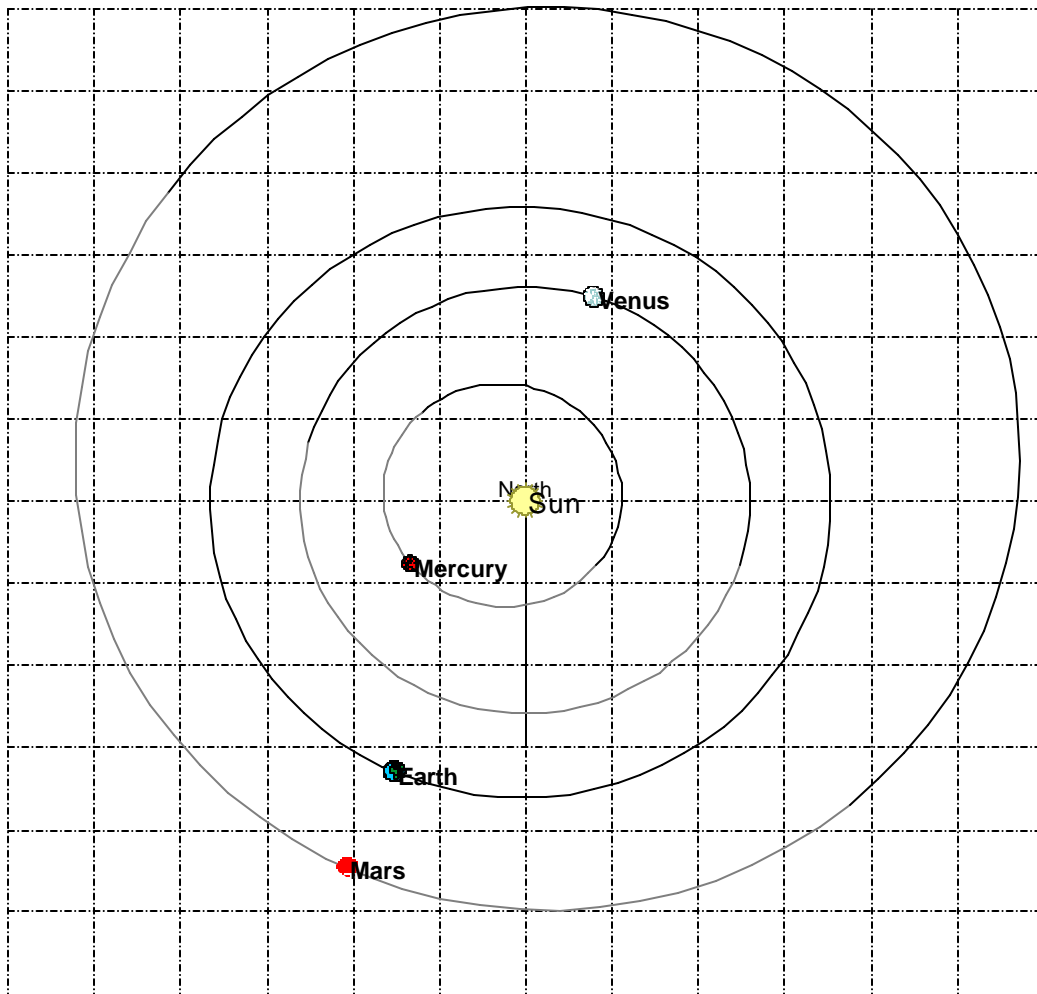
The "Orrery View" of the solar system on the next page shows that the opposition of 2003 is almost perfectly aligned with the perihelion of Mars. On the page following that the star field along the retrograde path of Mars is shown. Once again, as in 1672, Mars will be traveling through Aquarius, on a retrograde path very similar to its path then. But it will not come close to the three bright stars used by Cassini and Flamsteed this time. It comes within 31' of a magnitude 4 star on July 15, some six weeks prior to opposition (although separation is 31', the RA separation is very small, making it possible for us to measure with the micrometer eyepiece, whose field of view is about 50'). But after that Mars comes close to only magnitude 7 stars for the next 2-3 months. Cassini and Flamsteed were very lucky in 1672, as well as very talented!

With the improvements in apparatus and technique developed in 2001, we hope to re-measure the Astronomical Unit to within ±20% during the 2003 opposition.

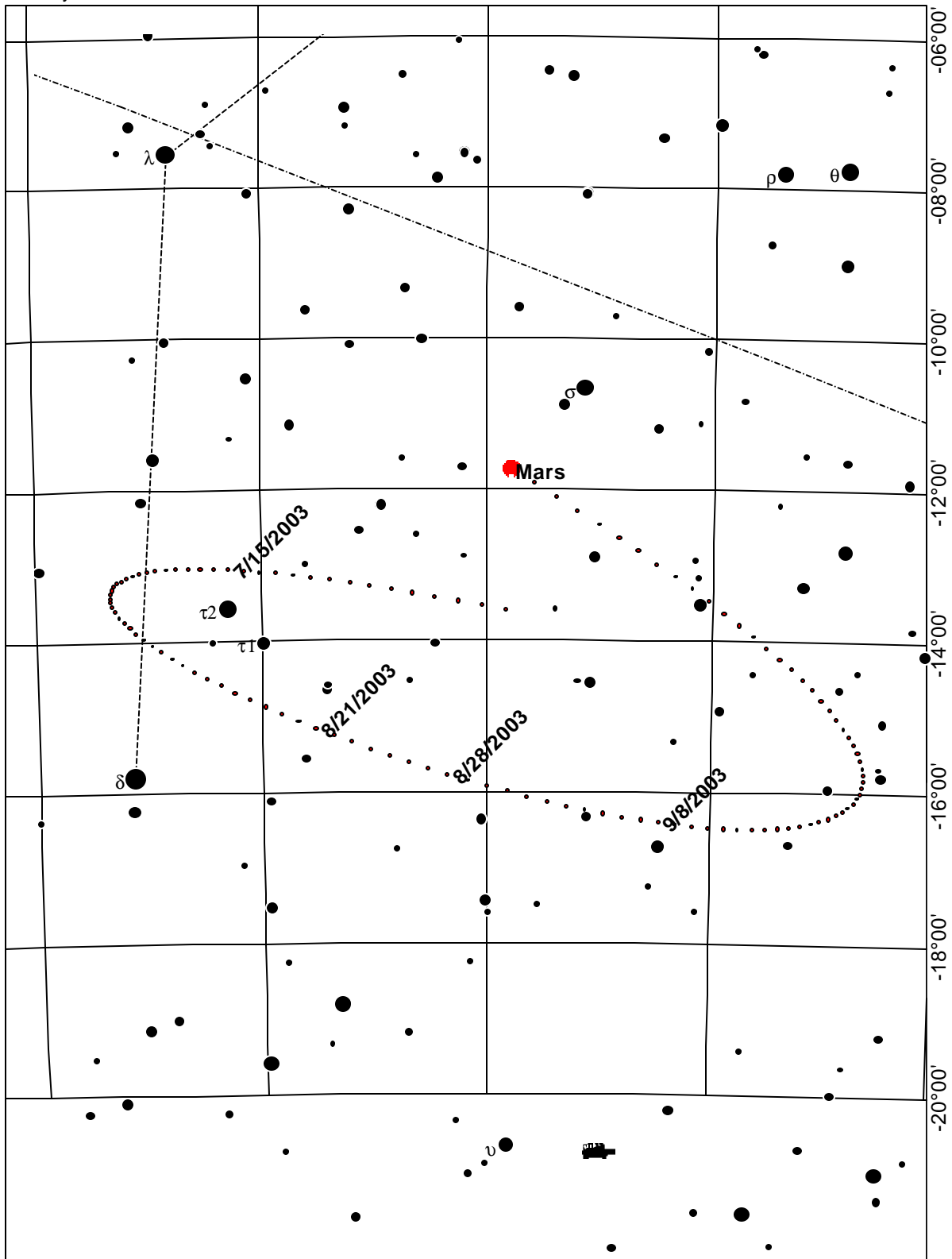References (can be found in the JJMO Robson Library):

Van Helden, Albert, "Measuring The Universe: Cosmic Dimensions from Aristarchus to Halley", 1985. A copy of chapters 9-12 is in the Robson Library. Chapter 12 is an excellent description of the 1672 measurements of Flamsteed, Cassini, and Richter.

Web, Stephen, "Measuring The Universe: The Cosmological Distance Ladder" 1999. Contains a good description of the Cassini and Flamsteed measurements.

"Orrery View" of the solar system at the 8/28/2003 Mars opposition.

Some measurement opportunities during the 2003 opposition of 8/28/2003.

Interpolated rate of Mars proper motion @ ~11:30 PM EDT 0n 6/27/01 = 46.0"/h ± 1.4%

Data Point	First meas. of delta RA (EDT)	2nd Meas. of delta RA (EDT)	Delta t (Hours)	α−β (arc sec)	Mars Proper Motion, (arc sec)	Parallax, p = α−β−θ (arc sec)	Chord (miles)	Dist. to Mars, d (miles)
1	22:08	0:57	2.8167	160.66	129.6	31.1	2134.0	1.42E+07
2	22:10	1:00	2.8333	144.04	130.3	13.7	2146.0	3.23E+07
3	22:12	1:04	2.8667	182.82	131.9	50.9	2170.1	8.79E+06
4	22:15	1:06	2.8500	146.81	131.1	15.7	2158.0	2.84E+07
5	22:17	1:09	2.8667	163.43	131.9	31.5	2170.1	1.42E+07
					error est.-> ±1.8"	28.6 <--- Avg.-->		1.96E+07
						13.4 <-- SD -->		9.10E+06
						±14" = 50% <--error est.		

Raw α−β measurements, using calibration of 277± 7 arc sec/turn from SAO 185033 and SAO 185020:

Data Point	First dial reading on: Mars (turns)	SAO185033 (turns)	α (turns)	α (arc sec)	Second dial reading on: Mars (turns)	SAO185033 (turns)	β (turns)	β (arc sec)	α−β (arc sec)
1	3.25	1.42	1.83	506.91	2.55	3.8	1.25	346.25	160.66
2	3.21	1.38	1.83	506.91	2.52	3.83	1.31	362.87	144.04
3	3.24	1.34	1.90	526.30	2.52	3.76	1.24	343.48	182.82
4	3.21	1.41	1.80	498.60	2.50	3.77	1.27	351.79	146.81
5	3.21	1.39	1.82	504.14	2.55	3.78	1.23	340.71	163.43
			508.6 <-------------- Average -------------->					349.02	159.55
			9.4 <--------- Standard Deviation -------->					7.2	13.86

Calc. of α−β using Horizons "airless apparant" option for Mars; RA(SAO 185033) = 17h06m58.953s

Data Point	α (arc sec)	β (arc sec)	α−β (arc sec)	Mars Proper Motion, (arc sec)	Parallax (arc sec)	= α − β − θ
1	534.75	392.70	142.05	129.6	12.48	
2	533.10	390.90	142.20	130.3	11.87	
3	531.30	387.60	143.70	131.9	11.83	
4	528.90	385.95	142.95	131.1	11.85	
5	527.25	383.55	143.70	131.9	11.83	
			142.90 ±0.8"			

N.B., RA separations decrease uniformly, as they should. Also, it appears that the dial calibration of 277"/turn is somewhat low. A more rigid, higher resolution motion control dial is needed.

For parallax of 11.85" and baseline of 2150 miles, dist. to Mars = 37.4 million miles

For dist to Mars of 42 million miles and baseline of 2150 miles, parallax should be 10.6"

PEM, rev.8/30/01